On the Motion of a Viscoelastic Fluid Through a Cylinder of Rectangular Cross Section

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Several statements to the contrary notwithstanding, if it exists, the steady rectilinear shearing motion postulated by Han (1971), that is,

$$v_x = 0,\ v_y = 0,\ v_z = f(x, y), \tag{1}$$

is most certainly a steady viscometric flow. Such a conclusion follows if the flow is viewed in the x^1, x^2, x^3 coordinate system where the x^1 coordinate surfaces are the cylinders $f(x, y) = \text{const.}$, the x^2 coordinate surfaces are the planes $z = \text{const.}$ and the x^3 coordinate surfaces are the cylinders $g(x, y) = \text{const.}$ where $\nabla g \cdot \nabla f = 0$. Thus, in the x^1, x^2, x^3 coordinate system Equation (1) becomes

$$v^1 = 0,\ v^2 = x^1,\ v^3 = 0$$

and the motion is seen to belong to the steady viscometric class.

The main issue, however, is whether or not a steady rectilinear shearing motion in a cylinder of rectangular cross section can be compatible with the dynamic equations which in view of Equation (1) take the special form

$$\underline{i}_z \cdot \nabla \cdot \underline{\underline{T}} = 0 \tag{2a}$$

$$(\underline{\underline{I}} - \underline{i}_z \underline{i}_z) \cdot \nabla \cdot \underline{\underline{T}} = 0 \tag{2b}$$

Thus, it is important to know the conditions under which two scalar valued functions $v_z = f(x, y)$ and $p = \alpha z + \phi(x, y)$ can be found such that Equations (2a) and (2b) are satisfied exactly.

With reference to Han such a determination need only be made for the class of fluids defined by the Williams and Bird (1962) simplification of the more general Oldroyd (1958) equation. Such fluids are defined by three material constants η_0, λ_1, and λ_2, in steady viscometric flows, their behavior is completely determined by three material functions $\mu(|\nabla f|)$, $\sigma_1(|\nabla f|)$, and $\sigma_2(|\nabla f|)$, Truesdell and Noll (1965). Rectilinear flow through a channel, a limiting configuration of the rectangular cylinder, is easily seen to be of the steady viscometric class when viewed using the rectangular coordinate system of Han. Thus, if $\partial v_z/\partial x = 0$ and $x^1 = y$, $x^2 = z$, $x^3 = x$, Han's Equations (9) to (14) yield the material functions for rectilinear channel flow, and thereby for all steady viscometric flows as

$$\mu = \eta_0 \frac{1 - \dfrac{4}{3}\lambda_1\lambda_2 |\nabla f|^2}{1 + \dfrac{1}{6}\lambda_1{}^2 |\nabla f|^2} \tag{3a}$$

$$\sigma_1 = \left(\frac{1}{2}\lambda_1\mu - 2\eta_0\lambda_2\right)|\nabla f|^2 \tag{3b}$$

and

$$\sigma_2 = 2\lambda_1\mu|\nabla f|^2 \tag{3c}$$

For flow in cylinders of arbitrary but constant cross section, Ericksen (1956) has shown that the condition

$$\frac{\sigma_1}{\mu|\nabla f|^2} = \text{const.} \tag{4}$$

is necessary and sufficient for the solution of Equation (2a) to be compatible with Equation (2b). It follows from Equation (4) that for steady rectilinear flows of the fluids studied by Han to be possible in cylinders of noncircular cross section the material parameter λ_2 must have the value zero. Unfortunately, the viscosity function μ is highly sensitive to the value of the parameter λ_2; for a given value of λ_1, λ_2 determines the asymptotic value of μ for large values of $|\nabla f|$, and thus for Equation (3a) to hold over a wide range of $|\nabla f|$, it follows that λ_2 must satisfy the conditions

$$\lambda_2 \neq 0, \quad \lambda_1\lambda_2 < 0.$$

Should experiment support the postulate embodied in Equation (1), then either the fluids are not of the class described by the Williams and Bird equation, or $v_{z\max}$ is sufficiently small that the secondary motion is not detectable. The latter is a likely happenstance in general inasmuch as every solution of Equation (2a) is compatible with Equation (2b) through the first three orders in $|\nabla f|$; in particular, secondary flows could go unnoticed in the low velocity experiments of Han. Nonetheless, it remains true that a rectilinear motion can not satisfy Equations (2a) and (2b) exactly, either for the Williams and Bird fluids or for more complex fluids; thus it is not clear which if any of Han's results can be believed in view of the inability of Williams and Bird fluids, with $\lambda_2 \neq 0$, to exhibit a balanced stress system in rectilinear shearing in noncircular cylinders. The above statement not only places in doubt the formula relating the velocity gradient to the normal stress differences and hence to the exit pressure measurements but is especially damaging to those conclusions drawn from the approximate solution, that is,

$$\frac{v_z}{v_{z\max}} = \left[1 - \left(\frac{x}{a}\right)^m\right]\left[1 - \left(\frac{y}{b}\right)^m\right]$$

of Equation (2a), when in fact the exact solution of Equation (2a) is not compatible with Equation (2b).

NOTATION

a, b = ½ width, ½ depth of rectangular cylinder
f, g = curvilinear coordinates in x, y plane
I = identity dyadic
m, n = parameters
p = pressure
T = stress dyadic
v_x, v_y, v_z = rectangular components of the velocity vector
v^1, v^2, v^3 = curvilinear components of the velocity vector
x, y, z = rectangular coordinates
x^1, x^2, x^3 = curvilinear and/or rectangular coordinates

Greek Letters

α = $\partial p/\partial z$
η_0, λ_1, λ_2 = material parameters

μ, σ_1, σ_2 = material functions
ϕ $= p - \alpha z$

LITERATURE CITED

Ericksen, J. L., *Quart. Appl. Math.*, **14**, 318 (1956).
Han, C. D., *AIChE J.*, **17**, 1418 (1971).
Oldroyd, J. G., *Proc. Roy. Soc.*, *A245*, 278 (1958).
Truesdell, C. A., W. Noll, *The Nonlinear Field Theories of Mechanics*, in "Encyclopedia of Physics," Vol. III/3, S. Flugge, ed., Springer-Verlag, Berlin (1965).
Williams, M. C., and R. B. Bird, *Physics of Fluids*, **5**, 1126 (1962).

Solid Particle Deposition From a Turbulent Gas Stream

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Friedlander and Johnstone (*1*) presented experimental data for the wall deposition of small iron and aluminum particles from a turbulent air stream. The data were compared with an analysis considering equal eddy diffusion of the particles and gas. The eddies were assumed to carry the particles to one stopping distance of the wall because of the relatively stagnant fluid layer at the wall. The distance that a particle with a given initial velocity will move through a stagnant fluid is defined as the stopping distance. This is represented by the equation

$$S = \frac{m}{3\pi\mu d_p} v_p \tag{1}$$

A transport equation was derived using the Lin, Moulton, and Putnam (*2*) eddy diffusion equations for the sublayer and transition region and with the assumption that the mass flux from the center line to the wall is equal to the mass flux at the wall. A particle velocity of $0.9u^*$ was assumed and solution of the derived equation showed agreement with the experimental data except for the 0.8 micron particles.

Hughmark (*3, 4*) has proposed diffusion equations for the sublayer and transition region which represent a higher eddy diffusion rate than is shown by the equations used by Friedlander and Johnstone. Earlier work (*5*) shows a numerical integration of equations for the turbulent flow velocity profile and eddy diffusion to calculate mass transfer for a smooth circular pipe. This calculation technique can be used with the eddy diffusion equations to determine mass transport in an air stream without the molecular diffusion contribution. The Friedlander and Johnstone experimental data for mass transfer can thus be used to estimate the distance from the pipe wall that corresponds to the observed mass transfer rate. This distance can then be compared with the stopping distance in accordance with Equation (1) to estimate the particle velocity. Figure 1 shows the dimensionless particle velocity as a function of the dimensionless stopping distance. Data for 0.8 micron particles in the 0.54 cm. diam. pipe are not shown because these data are not in agreement with the other particle data. The average value of $0.5u^*$ for the particle velocity corresponds to the radial fluctuating velocity at $y^+ \approx 12$ observed by Laufer (*6*). A wall region analysis (*4*) indicates that the source of the wall region fluctuations corresponds to $y^+ \approx 12$. Thus the assumptions of equal particle and gas diffusion, stopping distance, and particle velocity equal to fluctuating velocity appear to be consistent with the experimental data.

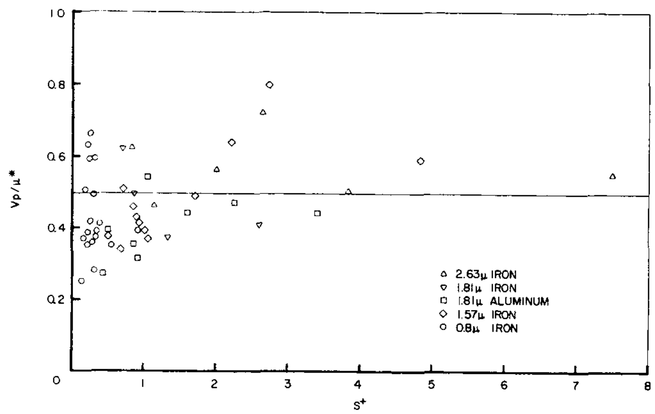

Fig. 1. Particle velocity.

NOTATION

d_p = particle diameter
f = friction factor
m = mass of particle
S = stopping distance
S^+ = reduced stopping distance, Su^*/ν
u^* = $v_{av}\sqrt{f/2}$
v_{av} = average gas velocity
v_p = particle velocity
y = distance from wall
y^+ = yu^*/ν
μ = gas viscosity
ν = kinematic viscosity

LITERATURE CITED

1. Friedlander, S. K., and H. F. Johnstone, *Ind. Eng. Chem.*, **49**, 1151 (1957).
2. Lin, C. S., R. W. Moulton, and G. L. Putnam, *ibid.*, **45**, 636 (1953).
3. Hughmark, G. A., *AIChE J.*, **17**, 51 (1971).
4. *Ibid.*, 902 (1971).
5. ———, *Ind. Eng. Chem. Fundamentals*, **8**, 31 (1969).
6. Laufer, J., Natl. Advisory Committee Aeron., Tech. Note 2954 (1953).